Quantum Field Theories and Representation Theory

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Organized by
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Abstract

We report on the international workshop "Quantum Field Theories and Representation Theory" which was held at Osaka Metropolitan University during March 27–30, 2023. The workshop was focused on the interplay of quantum field theories (QFTs) in physics and representation theory in mathematics, and involve eleven invited talks and nine contributed talks. The topics of the workshop include non-perturbative aspects of QFTs, supersymmetry and superconformal symmetry, vertex algebras associated with QFTs, Coulomb branch of 3D supersymmetric QFTs, and algebraic structures arising from string theory and M-theory.

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Preface

Quantum field theory and representation theory are two major subjects in physics and mathematics, whose recent developments are closely related to each other. This is partially because supersymmetric quantum field theories in various dimensions provide many algebraic structures and their representations. The aim of the workshop "Quantum Field Theories and Representation Theory", which was held at Osaka Metropolitan University in March 2023, was to bring together experts in supersymmetric quantum field theories and those in representation theory to discuss recent advances in these subjects and find new directions of research.

This workshop was focused on the interplay of quantum field theories (QFTs) and representation theory, and covered the following topics:

- Non-perturbative aspects of QFTs
- Supersymmetry and superconformal symmetry
- Vertex algebras associated with QFTs
- Coulomb branch of 3D supersymmetric QFTs
- Algebraic structures arising from string theory and M-theory

We had eleven invited talks and nine contributed talks on these and related topics, which provided us with a clear and wide understanding of the recent developments in the study of QFTs and representation theory. As a result, it turned out that different problems on different topics often share some characteristics so that a similar analysis or technique can be applied to them. It also happened in some cases that seemingly different approaches in math and physics describe one common phenomenon. These strongly suggest that representation theory and QFT are still developping together in various directions. In the main part of this report, we collect the titles and abstracts of the invited and contributed talks.

In addition to the above success in the scientific sense, we also succeeded in supporting young researchers. Indeed, many of the contributed talks was given by young researchers and postdocs from Japan and abroad. We hope that this will encourage them and promote their career development, which would then lead to strong research activities in the fields of representation theory and quantum field theories in the future.

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On Classification of Fermionic Rational CFT

Sungjay Lee

(Korea Institute for Advanced Study)

We systematically study how the integrality of the conformal characters shapes the space of fermionic rational conformal field theories in two dimensions. The integrality suggests that conformal characters on torus with a given choice of spin structures should be invariant under a principal congruence subgroup of $PSL(2, Z)$. The invariance strongly constrains the possible values of the central charge as well as the conformal weights in both Neveu-Schwarz and Ramond sectors, which improves the conventional holomorphic modular bootstrap method in a significant manner. This allows us to make much progress on the classification of fermionic rational conformal field theories with the number of independent characters less than five.

Invertible giant graviton expansions

Yosuke Imamura

(Tokyo Institute of Technology)

We study some examples of giant graviton expansions of the superconformal index (SCI). In particular, we focus on the expansions with simple sum. In general, the simple-sum giant graviton expansion expresses the SCI of a theory $A[N]$ with rank N as a series of the SCI of another set of theories $B[m]$ with rank m. We find that this relation is invertible, meaning that the SCI of $B[m]$ can also be expressed as a series of the SCI of $A[N]$.

K theoretic instanton counting with a defect and qq-Painlevé VI equation

Hiroaki Kanno

(Nagoya University)

We argue that the non-stationary difference equation proposed by S.Shakirov [arXiv 2111.07939] is identified with a quantization of the discrete version of Painlevé VI equation, whose time evolution is generated by a translation element in the extended affine Well group of $D_5^{(1)}$. Accordingly, the original difference equation can be factorized into a coupled system. We conjecture that solutions to the coupled system are given by the instanton partition function of the five dimensional SU(2) gauge theory with a surface defect. The talk is based on a collaboration with H.Awata, K.Hasegawa, R.Ohkawa, S.Shakirov, J.Shiraishi and Y.Yamada, arXiv [2211.16772].

Solitons in 4-dim. WZW Model

Masashi Hamanaka

(Nagoya University)

Four-dimensional Wess-Zumino-Witten (WZW_4) models are analogous to the two dimensional WZW models. Equation of motion of the WZW_4 model is the Yang equation which is equivalent to the anti-self-dual Yang-Mills equation. In the split signature $(+,+,-,-)$, the WZW_4 model describes the open $N = 2$ string theory in the four-dimensional space-time. In this talk, we discuss soliton-type classical solutions of the WZW_4 model (mainly in the split signature) by calculating the action density of the WZW_4 model. We find that the action density of the one-soliton solutions is localized on a three-dimensional hyperplane. This shows that there would be codimension-one-solitonic objects, or equivalently, some kind of three-branes in the open $N = 2$ string theory. We also prove that in the asymptotic region of the space-time, the action density of the n-soliton solutions is a "nonlinear superposition" of n one-solitons. This suggests the existence of intersecting n three-branes in the $N = 2$ strings. Finally we make a reduction to $(1+2)$-dimensional real spaces to calculate energy densities of the soliton solutions. We can successfully evaluate the energy distribution for the two-soliton solutions and find that there is no singularity in the interacting region. This implies the existence of smooth intersecting codimension-one branes in the whole region. Soliton solutions and instanton solutions in the Euclidean signature are also discussed if time permits.

Tensionless limit of $\mathbf{AdS_3/CFT_2}$ from integrability

Ryo Suzuki

(Shing-Tung Yau Center of Southeast University)

The $AdS_3 \times S^3 \times T^4$ spacetime with pure Ramond-Ramond flux, related to the D1-D5 system, is believed to be integrable as in $AdS_5 \times S^5$. The integrability approaches to AdS_3/CFT_2 and the mirror TBA equations to study the energy spectrum are reviewed. By solving the TBA for massless excited states at small coupling, we find that the leading-order contribution to the anomalous dimensions includes massless wrapping corrections. This talk is based on arXiv:2303.02120 with Alberto Brollo, Dennis le Plat and Alessandro Sfondrini.

New non-supersymmetric heterotic string theory with reduced rank and exponential suppression of the cosmological constant

Sota Nakajima

(KEK)

We study the heterotic asymmetric orbifold model in which supersymmetry is broken by the stringy Schark-Schwarz mechanism. This model is a natural non-supersymmetric extension of CHL strings and can also be interpreted as the interpolating model between the $E_8 \times E_8$ theory and the non-supersymmetric E_8 theory. The enhancement of gauge groups, of which the rank is reduced to $8+d$, is explored. In particular, the enhancement to non-simply-laced groups is possible with $d \geq 2$, as well as in the CHL model. We also give the conditions that the massless matter spectrum must satisfy. Moreover, the one-loop cosmological constant is evaluated in the regime where supersymmetry is asymptotically restored, and we show that the exponential suppression can occur unless $d = 1$.

Twisting 6d SCFTs and LSTs

Kimyeong Lee

(Korea Institute for Advanced Study)

Vertex algebra extensions and SQFT on the interval

Mykola Dedushenko

(SCGP, Stony Brook University)

Boundary conditions B preserving $(0,2)$ supersymmetry in three-dimensional $\mathcal{N}=2$ supersymmetric QFT (SQFT) support boundary vertex algebras $VA[B]$. When a theory is put on an interval, the $\mathcal{N}=(0,2)$ boundary conditions B_1 and B_2 support the boundary chiral algebras $VA[B_1]$ and $VA[B_2]$, and the total chiral algebra is a VOA obtained by extending $VA[B_1]\otimes VA[B_2]$ by their bimodules. The latter correspond to SUSY line operators extended between the boundaries. I will discuss an example of this construction related to chiral differential operators on the group, with some puzzles and applications. Based on joint papers: one with M. Litvinov, and one with M. Litvinov and S. Alekseev.

On localization of Higgs branch VOAs

Tomoyuki Arakawa

(Research Institute for Mathematical Science, Kyoto University)

On the Beem-Nair Conjecture

Shun Furihata

(Research Institute for Mathematical Science, Kyoto University)

Given a simple Lie group G, we have an open immersion (constructed by Beem and Nair) from the Kostant-Toda lattice associated to G into the universal centralizer of G. They expected that a free field realization of the chiral universal centralizer of G at the critical level will be obtained by the chiralization of this immersion. In this talk, we will verify this conjecture is true by constructing an embedding from the chiral universal centralizer into an appropriate vertex operator algebra at any level.

On intermediate Lie algebra $E_{7\frac{1}{2}}$

Kaiwen Sun

(Korea Institute for Advanced Study)

$E_{7\frac{1}{2}}$ is an intermediate Lie algebra filling the hole between E_7 and E_8 in the Deligne exceptional series. It was found independently by Mathur, Muhki, Sen in the modular bootstrap of 2d rational CFTs and by Deligne, Cohen, de Man in representation theory. We propose the VOA associated with $E_{7\frac{1}{2}}$ at level 2 and the VOA associated with 4d rank-two instanton SCFT of $E_{7\frac{1}{2}}$. We compute their characters and provide some coset constructions. These generalize the previous works of Kawasetsu for $E_{7\frac{1}{2}}$ at level 1 and of Arakawa-Kawasetsu at level -5. This is based on a joint work with Kimyeong Lee and Haowu Wang.

S^1 Reduction of 4D $\mathcal{N} = 3$ SCFTs and Squahing Independence of ABJM Theories

Tomoki Nakanishi

(Osaka Metropolitan University)

We study the compactification of 4D $\mathcal{N} = 3$ superconformal field theories (SCFTs) on S^1, focusing on the relation between the 4D superconformal index and 3D partition function on the squashed sphere S^3_b. Since the center $\mathfrak{u}(1)$ of the $\mathfrak{u}(3)$ R-symmetry of the 4D theory can mix with an $\mathcal{N} = 6$ abelian flavor symmetry in three dimensions, the precise 4D/3D relation for the global symmetry is not obvious. Focusing on the case in which the 3D theory is the ABJM theory we demonstrate that the above R-symmetry mixing can be precisely identified by considering the Schur limit (and/or its $\mathcal{N} = 3$ cousin) of the 4D index. As a result, we generalize to the ABJM theories recent discussions on the connection between supersymmetry enhancement of the 4D index and squashing independence of the S^3_b partition function.

3d Seiberg-like duality and quantum K-theory

Yutaka Yoshida

(Meiji Gaukuin University)

In this talk, I will discuss the properties of quantum K-theory of the moduli space of the Higgs branch vacua predicted by the Seiberg-like duality of the Chern-Simons-matter theories. When two varieties are realized as the moduli spaces of a Seiberg-like dual pair, the K-theoretic I-functions with the level structure of these varieties satisfy a relation derived from the wall-crossing formula of indices of handsaw quiver varieties.

Coulomb branches and DAHA

Hiraku Nakajima

(Kavli Institute for the Physics and Mathematics of the Universe)

I will review a mathematical approach to Coulomb branches of 3d $\mathcal{N} = 4$ theory in my joint work with Braverman and Finkelberg. Then I will explain DAHA as an example of a quantization of Coulomb branch. If time permits, I would like to explain one new example.

On Holonomy Saddles

Piljin Yi

(Korea Institute for Advanced Study)

I overview the holonomy saddle and explore a few classes of theories where the notion offers crucial tools and cautionary tales when gauge theories are defined in spacetimes with nontrivial topology. Discussion will include (de)confinement transition, refined Witten indices, A-twisted susy partition functions, and 4d/5d relation of Seiberg-Witten theories. For the last example, we also give reconstruction of 5d BPS quivers via the holonomy saddles.

Higgs branch of $Dp - O(p+2)^+$ brane systems

Mohammad Akhond

(Kyoto University)

Crystallographic complex reflection groups, 4d $\mathcal{N} = 2$ SCFTs, and Seiberg-Witten integrability

Yongchao Lu

(Korea Institute for Advanced Study)

We study various classes of 4d $\mathcal{N} = 2$ SCFTs associated with crystallographic complex reflection groups (CCRG). We propose that the geometry of their Coulomb branch is captured by elliptic integrable systems of the Calogero-Moser type attached to CCRG. Using elliptic Cherednick algebra and elliptic Dunkl operators, we can derive the Lax matrix, spectral curve, and their quantizations. This talk will center on Mininan-Nemeschansky theories and their higher-rank generalizations, as well as their connection with Hitchin systems through class S construction and 5d quantum curves via circle compactification.

Root of unity limit and elliptic deformation of q-Virasoro block

Reiji Yoshioka

(Osaka Central Advanced Mathematical Institute, Osaka Metropolitan University)

In the so-called AGT correspondence, the key object on the CFT side is the Virasoro (conformal) block. The q-Virasoro block is its q-analog, and it is known that the corresponding object is the 5-dimensional instanton partition function. In this talk, the root of unity limit and the elliptic deformation of q-Virasoro block will be discussed. The purpose of this talk is to provide an explicit formulation of these. I will show that the super Virasoro algebra appears in the root of unity limit, on the other hand the elliptic Virasoro block can be interpreted as a q-Virasoro block at finite temperature.

Duality of adjoint SQCD with Argyres-Douglas blocks

Kazunobu Maruyoshi

(Seikei University)

In this talk we present a dual description of $\mathcal{N} = 1$ $SU(N)$ gauge theory coupled to N_f pairs of fundamental and anti-fundamental chiral multiplets (N_f flavors) and to $D_p[SU(N)]$ theory of Argyres-Douglas type with a certain superpotential deformation, which is $\mathcal{N} = 1$ adjoint $SU(N)$ SQCD deformed by Tr ϕ^{p+1} with N_f flavors. We perform a few checks of this conjecture by the operator matching and by the matching of superconformal indices. We also generalize this to the duality between $\mathcal{N} = 1$ $SU(N)$ gauge theory with two $D_p[SU(N)]$ theory and $\mathcal{N} = 1$ $SU(N)$ gauge theory with two adjoint chiral multiplets.

M2-branes —Parallelotopes and Bilinear Relations—

Sanefumi Moriyama

(Osaka Metropolitan University)

M2-branes are closely related to duality cascades. From physical questions on the finiteness and the uniqueness of duality cascades, we propose that the fundamental domain of duality cascades is a parallelotope, which is a polytope tiling the whole space by translations. For the special case with symmetries of the Weyl group, the parallelotope is the affine Weyl chamber enjoying hidden symmetries of the affine Weyl group. This implies that physical quantities of the system satisfy the q-Painlevé equation. I will explain these aspects of M2-branes in my talk.