Helicity and space-time symmetry: a new perspective of classical and quantum systems

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Helicity and space-time symmetry — a new perspective of classical and quantum systems

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Abstract

The helicity is a key ingredient of the Nambu-bracket for the Euler equation for motion of an ideal fluid. This conference focuses on the helicity, as the common structure of the non-canonical Hamilton structure and the Nambu bracket for the ideal fluid motion, and deepens its mathematical and physical significance that transcends the subjects, by enlightening the symmetry behind its preservation. With interdisciplinary fusion, we explore a new theoretical framework and seek new applications of the helicity.

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Preface

"Helicity" is an integral invariant characterizing vortex structures that appear in a variety of systems, from quantum (micro) to classical (macro) fields. Since a vortex, which is a chiral structure with a relatively long life, has odd parity, its generation cannot be explained by the energy (Hamiltonian) alone, and helicity is the key. In the joint use / joint research conducted by the proposer as a representative in FY2020, with the Nambu brackets as the theme, we collected classical and quantum systems in which vortices and string-like structures play major roles, in order to explicitly capture the helicity in the non-canonical Hamiltonian structure, and put in perspective their phenomena and theories that describes the underlying fields. It became clear that formulation based on the Nambu brackets is yet completed, and that there is a gap between its formulation and the framework of the non-canonical Hamiltonian system. In this research, we focus on the helicity, which is located in the common part of the non-canonical Hamiltonian structure and the Nambu mechanics, and deeply dig into the helicity as a basic concept that transcends the subjects through elucidation of the symmetries behind the phenomena. The purpose was to explore a new theoretical framework of the helicity and further to explore its application.

The helicity was originally introduced as the inner product of momentum and spin in the relativistic quantum mechanics. The fluid helicity defined by the integral of the vorticity field and the velocity field is its macroscopic counterpart, the topological meaning of which was clarified by Moffatt (1969). The difference between the two can be attributed to the difference in the hierarchy level, but there remains a feeling of oversight from the viewpoint of the chiral structure. The dynamics of a macroscopic system is described by relationships among the vector (tensor) fields and their derivatives (the Eulerian description), disregarding the particle view (the Lagrangian description). The helicity resides in a common place of the Eulerian and the Lagrangian descriptions. The aim of this research was to bridge the macro- and micro-systems, such as classical and quantum fluids, BEC, plasma, crystals, and optics, by delving into the helicity from the viewpoint of space-time symmetry.

In recent years, there have been a series of interesting discoveries associated with the helicity. The Jones and HOMFLYPT polynomials, which are quantum invariants of knots and links, were derived from fluid helicity (Liu & Ricca 2016), an experimental realization of knots and links of vortex filaments in a fluid (Irvine 2012), possibility of space-time curvature in general relativity for the origin of the cosmic magnetic field and the contribution of the relativistic helicity (Mahajan & Yoshida 2010), the generation of chiral nanostructures of materials by laser ablation based on the spin angular momentum of light (Toyoda *et al.* 2013), etc. We also discussed a wide range of applications of the helicity, from mathematics to engineering.

Organizers

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A description of perfect fluid by Kalb-Ramond field and the duality with Gross-Pitaevskii

Y. Matsuo (U. Tokyo)
A. Sugamoto (Ochanomizu U.)

In this talk, we explain a description of the perfect fluid in terms of a two-form gauge field (Kalb-Ramond field). Nambu proposed such an idea long ago in 1977, and one of the authors (AS) developed the idea further.

In the perfect fluid motion, the vortex keeps the circulation and behaves as a "soliton". In this sense, it is natural to describe the fluid dynamics in terms of the vortices, which were explored long ago by Lund and Regge.

The Kalb-Ramond field couples naturally with the vortex degree of freedom through the integration over the vortex (string) world-sheet. We study the detailed relation between the dynamical degree of freedom of the Euler equation with the Kalb-Ramond field.

We also compare it with the Gross-Pitaevskii equation, giving an alternative description of the fluid motion. We explain that the two descriptions are dual, similar to the electric/magnetic fields in the Maxwell theory. In this sense, we expect to have the strong/weak duality in the two descriptions.

The kinetic origin of the fluid helicity — a symmetry in the kinetic phase space

Z. Yoshida (Natl. Inst. Fusion Sci.)
P. J. Morrison (U. Texas, Austin)

Helicity, a topological degree that measures the winding and linking of vortex lines, is preserved by ideal (barotropic) fluid dynamics. In the context of the Hamiltonian description, the helicity is a Casimir invariant characterizing a foliation of the associated Poisson manifold. Casimir invariants are special invariants that depend on the Poisson bracket, not on the particular choice of the Hamiltonian. The total mass (or particle number) is another Casimir invariant, whose invariance guarantees the mass (particle) conservation (independent of any specific choice of the Hamiltonian). In a kinetic description (e.g. that of the Vlasov equation), the helicity is no longer an invariant (although the total mass remains a Casimir of the Vlasov's Poisson algebra). The implication is that some "kinetic effect" can violate the constancy of the helicity. To elucidate how the helicity constraint emerges or submerges, we examine the fluid reduction of the Vlasov system; the fluid (macroscopic) system is a "sub-algebra" of the kinetic (microscopic) Vlasov system. In the Vlasov system, the helicity can be conserved, if a special *helicity symmetry* condition holds. To put it another way, breaking helicity symmetry induces a change in the helicity. We delineate the geometrical meaning of helicity symmetry, and show that, for a special class of flows (so-called epi-2 dimensional flows), the helicity symmetry is written as $\partial_\gamma = 0$ for a coordinate γ of the configuration space.

Invariance of helicity and some topological implications

H. Keith Moffatt (Trinity College, Cambridge)

Helicity is a topological invariant of any solenoidal vector field that is transported by (i.e. 'frozen in') a fluid flow. The best known examples are the magnetic field in a perfectly conducting fluid and the vorticity field in an ideal fluid, the latter providing an important invariant of the classical Euler equations of fluid flow. In this lecture, I shall first discuss the historical origin of these invariants, and I will then comment on two complementary problems in which helicity plays a key role: (i) the spontaneous generation of magnetic field (i.e. dynamo action) by turbulence with non-zero mean helicity; and (ii) the relaxation of knotted magnetic flux tubes in a perfectly conducting fluid to minimum-energy states constrained by their topology. Here there is an analogy with the 'tight-knot' problem: what is the minimum length of rope of unit circular cross-section with which a knot of particular type may be tied? There are differences however in that, for the magnetic relaxation problem, it is the volume of the flux tube that is conserved under deformation (and not the area of its cross-section), and the internal twist of the magnetic field affects the minimum-energy state. A particular problem here concerns the possibility of two or more minimum-energy states for a given knot type, and the manner in which a transition from one such state to another may be accomplished. Some aspects of this problem, under current consideration, will be discussed, insofar as time permits.

Helicity and topology of a tangle of vortices

Carlo F. Barenghi (Newcastle U.)

We address the question as to whether turbulence contains vortex knots in a context which is simpler than ordinary fluids: the context of superfluid helium. Unlike what happens in ordinary Navier-Stokes fluids (where vorticity is a continuous field), in superfluid helium quantum mechanics constraints the vorticity to individual (discrete) vortex lines. Quantum turbulence is therefore a disordered tangle of vortex lines. Since the vortex core thickness is many orders of magnitude smaller than the average distance between vortex lines in typical experiments, the vortex tangle can be modelled using the classical Biot-Savart law and the assumption that vortex lines reconnect [1] when they collide (as seen in the experiments), a mesoscale level of description called the Vortex Filament Model (VFM).

After numerically generating a state of turbulence in a statistical-steady state, we exploit the discrete nature of the vorticity to instantaneously associate an Alexander polynomial to each vortex loop. The degrees of the Alexander polynomials can thus be used to quantify the complexity of the turbulence, revealing a highly knotted distribution [3].

In the last part of the lecture I shall discuss the classical definition of helicity [2] in the context of super fluid helium. In particular, I shall show that the classical definition of helicity is consistent with what we know microscopically about the super fluid vortex core from N-body quantum mechanics, without the difficulties which arise in the simpler mean-field approach of Gross-Pitaevskii theory. I shall show that helicity, compute in the VFM, captures the physical difference [4] between the two forms of quantum turbulence which have been identified in experiments and numerical simulations.

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Chiral radiation transport theory of neutrinos

N. Yamamoto (Keio U.)
Di-Lun Yang (Academia Sinica)

One of the most important properties of neutrinos in the Standard Model of particle physics is the chirality. Although neutrinos are considered to play important roles in the dynamics of core-collapse supernovae, this property has been completely neglected in the conventional neutrino transport theory. Recently, we have systematically constructed the radiation transport theory incorporating the effects of chirality of neutrinos based on the underlying quantum field theory. This is the chiral radiation transport theory [1].

In this talk, we first give a brief introduction to transport phenomena due to the chirality of elementary particles, called chiral transport phenomena. One of its consequences is the chiral plasma instability, which generates a magnetic field with magnetic helicity from the chirality imbalance of fermions. We then discuss applications of the chiral radiation transport theory for neutrinos in corecollapse supernovae, and in particular, its possible implications for the origins of magnetars and pulsar kicks [2].

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Gravitational waves caused by chiral transport phenomena in neutron stars and supernovae

S. Hanai (Keio U.)
N. Yamamoto (Keio U.)

Exploring the internal structure of neutron stars is an important issue in astrophysics. Asteroseismology is a method to study the inside of stars by analyzing their seismic oscillations. These oscillations are classifed according to their physical origins, such as p-modes due to pressure, g-modes due to buoyancy, and so on. In the case of neutron stars, gravitational waves are expected to be emitted induced by these seismic oscillations.

In recent years, chiral transport phenomena caused by the chirality of relativistic quarks have been vigorously discussed in the context of quark-gluon plasmas created in relativistic heavy-ion collisions experiments. Typical examples are the chiral magnetic effect, which is the current in the direction of the magnetic feld, and the chiral vortical effect, which is the current in the direction of the vorticity. It is known that these chiral transport phenomena induce collective excitations called chiral magnetic waves and chiral vortical waves, respectively.

In this talk, we will show that chiral magnetic waves are generated in the electron matter inside neutron stars, and examine their properties. In particular, we will discuss the effect of electron mass on the chiral waves. We will estimate the frequency of new types of gravitational waves produced by these chiral magnetic waves and the possibility of observations. We will also show the possible existence of the other type of gravitational waves induced by the propagation of chiral vortical waves in the core of supernovae.

Convection and dynamo in newly-born neutron stars

Y. Masada (Aichi U. Edu.)
T. Takiwaki (Natl. Astron. Obs. Japan)
K. Kotake (Fukuoka U.)

Neutron stars (NSs) have the most extreme magnetic field in the universe, typically trillion, up to quadrillion times more powerful than Earth's. Although, we know, they are formed as an aftermath of massive stellar core-collapse, the origin of the magnetic field is still an outstanding issue in astrophysics. Mainly, two possible origins have been proposed: fossil field and dynamo field hypotheses. While the former regards it as an inheritance from NS's main sequence progenitor, the later presumes that it would be generated by some dynamo processes in newly-born NSs, also known as proto-neutron stars (PNSs). Although significant progress has been made in the study of the PNS dynamo, the origin of the diversity of NS's magnetic fields remains to be solved. "*What physics is responsible for the diversity of NS's magnetic fields ?*" To answer this question, we are now studying the properties of the convection and resultant dynamo in the PNS with the aid of numerical simulation. In this talk, we will present the results of our latest PNS dynamo simulations and the physics behind them, and also discuss the Beltrami structure of the flow and magnetic field, which is a natural consequence of the dynamo process.

Untying pathways of vortex knots

K. Shimokawa (Saitama U.)

In this talk we will analyze untying pathways of vortex knots by applying knot theory. The topology of entangled vortices is represented using oriented knots and links, and the reconnection of vortices is modeled using a coherent band surgery of knots and links. For the case where the topology of the vortex is a torus knot or link T(2,p), we characterize the untying pathway, and determine where the reconnection occurs.

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Physics of optical vortex and its application

R. Morita (Hokkaido U.)

Optical vortices (OVs), such as Laguerre-Gaussian modes, whose spatial phase depends on the azimuthal coordinate, have attracted much attention because of their unique properties. Possessing a phase singularity on their beam center, they carry the orbital angular momentum proportional to the topological charge, which is expressed by the value of the line integral of the spatial phase around the singular point. Thanks to these properties, they are used in the various applications such as laser processing, laser trapping/manipulation, quantum information processing, classical communications, super-resolution microscopy, and nonlinear spectroscopy. In the presentation, giving an overview of physical properties of OVs, we will discuss the interaction between OVs and materials from the viewpoint of orbital angular momentum, spin angular momentum, total angular momentum and helicity.

Detecting topology of fluid vortex knots in terms of polynomial invariants constructed from helicity

X. Liu (Beijing U. Tech.)
R. L. Ricca (U. Milano-Bicocca)

Helicity is the most important topological invariant in fluid mechanics, a conserved quantity for the Euler equation and the only integral invariant for volume-preserving diffeomorphic transformations [1]. From vortex filaments viewpoint, helicity provides access to characterize fluid knot topology. In the seminal work of Moffatt [2], then extended by Moffatt & Ricca [3][4], the relationship between helicity and knot linking numbers was revealed; since then linking number interpretation of helicity has played a key role in the study of the topological hierarchy of entangled fluid vortex knots. With the discovery of highly complex knots in superfluid turbulence [5] linking numbers and crossing numbers are proven to be no longer sufficient to detect topological complexity and changes in topology due to reconnections [6]. More powerful tools from knot theory are thus needed; for this reason we have derived Jones and HOMFLYPT knot polynomials from helicity [7][8]. In this talk we present a brief introduction to our derivation, emphasizing the role played by writhe and twist helicity in the polynomial skein relations.

This is joint work with Renzo L. Ricca (U Milano-Bicocca, Italy).

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Minimal unlinking pathways as geodesics in knot polynomial space

R. L. Ricca (U. Milano-Bicocca)
X. Liu (Beijing U. Tech.)
X.-F. Li (Guangxi U. Sci. Tech.)

During evolution fluid structures, such as linked and knotted vortex filaments in classical or quantum systems, decay naturally by a series of reconnection events through a process of consecutive topological simplifications. The process bears similarities with the DNA unlinking scenario studied in recombinant DNA plasmids [1]. By applying adapted knot polynomials, such as Jones and HOMFLYPT derived from helicity [2,3], we show that a topological cascade can be described as a geodesic flow in an appropriate knot polynomial space [4]. Each point of this space coincides with the knot type given by the polynomial value, that for moderate crossing number (≤ 9) identifies uniquely knot types. For the sake of example we consider adapted Jones polynomials with metric given by the inner product of Legendre polynomials. Unlinking pathways can thus be identified with geodesics on this space, and optimal decay paths measured by associated probability. Results are very promising, showing an interesting connection with the logarithmic behavior associated with the groundstate energy spectrum of tight knots and links [5].

This is joint work with Xin LIU (BJUT, China) and Xinfei LI (Guangxi UST, China).

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Review of Chern-Simons modified gravity — its effects on compact objects

K. Konno (Natl. Inst. Tech. Tomakomai College)

In recent years, many modified gravity theories have been investigated to seek the ultimate gravitational theory. Among them, Chern-Simons modified gravity is strongly suggested as an effective theory by two promising quantum gravity theories and by an extension of the standard model. Furthermore, Chern-Simons modified gravity is consistent with the recent results of gravitational wave detection.

In the present talk, I will briefly review Chern-Simons modified gravity from the fundamental part of this theory to recent topics. Chern-Simons corrections play an important role around rotating compact objects. Thus I focus on the effects of Chern-Simons modified gravity on compact objects in this talk.

Energy extraction from a rotating black hole

T. Igata (KEK)

No material body or light ray never can be extracted from a black hole. However, Penrose found that energy can be extracted from a rotating black hole [1]. In this talk, I will introduce an energy extraction mechanism from a rotating black hole that reconciles these seemingly contradictory statements (see, e.g., [2] for a review). The energy extraction mechanism mediated by a free electromagnetic field, the Blandford-Znajek process [3], has been widely believed to be a viable mechanism for relativistic jet formation because it achieves a large energy flux. From a spacetime perspective, the dynamics of magnetic field lines in a force-free magnetic field can be identified with the dynamics of strings [4, 5]. Using this fact, we will argue that the physical picture of the Blandford-Znajek process by a stationary axisymmetric force-free electromagnetic field is equivalent to the energy extraction process by a rigidly rotating Nambu-Goto string [6].

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Surface defects, vortices and integrable systems

K. Maruyoshi (Seikei U.)

In this talk, we discuss surface defects in a certain class of four-dimensional supersymmetric gauge theories, and a related vortex solution, in relation with the integrable lattice model. A surface defect has a support on a two-dimensional surface in the space-time. A typical surface defect is defined by the coupling with two-dimensional field theory on the surface or by the singular condition on the gauge field around the origin of the perpendicular direction. It is known that such a surface defect in four-dimensional supersymmetric gauge theory is realized as the low energy limit of the vortex solution of certain U(1) gauge theory associated with the original four-dimensional theory. By using this construction, we compute the partition functions of the supersymmetric gauge theory with the surface defect. It turns out that certain partition functions are identified with the transfer matrices of some integrable lattice models. We argue the reasons of this correspondence, and its application. This talk is based on the collaboration with Junya Yagi, and on arXiv:1606.01041, and the collaboration with Toshihiro Ota and Masahito Yamazaki, work in progress.

Richtmyer-Meshkov instability and vortices

K. Nishihara (Osaka U.),
C. Matsuoka (Osaka City U.)
T. Sano (Osaka U.)
F. Cobos-Campos (U. Castilla-La Mancha)
Vasily Zhakhovsky (Rosatom)

When an incident shock wave hits a corrugated interface, ripples are induced on the transmitted and reflected shocks. Due to the refraction of fluids crossing the rippled shock fronts, the velocity shear left at the interface behind the shocks. The Richtmyer-Meshkov instability occurs due to the shear flow at the corrugated interface. In linear regime, i.e., the distances among the interface and shock fronts are not large, the rippled shock fronts and the corrugated interface interact each other thorough sound wave and entropy wave. However it is known that the shock ripples and sound wave decay exponentially and the perturbation becomes incompressible. There exists an asymptotic linear growth rate. But fluids are not irrigational and vortices remain in the bulk between the shocks [1,2].

We investigated the nonlinear dynamics of RMI by developing a non-uniform vortex sheet model [3]. In RMI, bulk vortices are generated behind the rippled shocks. We here consider more generally nonlinear interactions between a non-uniform vortex sheet with point vortices. It will be shown that the nonlinear interaction between a non-uniform vortex sheet and point vortices as unstable and the mixing will be enhanced [4].

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Laser-driven interfacial instabilities in magnetized plasmas

T. Sano (Osaka U.)

Extreme plasma states observed in astrophysical phenomena can be created in the laboratory using large laser facilities such as the GEKKO laser at Osaka University. For example, the verification of the metallic hydrogen state under ultra-high pressure comparable to that of Jupiter's interior and the formation of collisionless shocks are being actively pursued. We are conducting laser experiments and MHD simulations on interfacial instabilities in magnetized plasmas associated with supernova explosions. Such phenomena are very important processes in laser fusion plasmas to realize ideal implosion. Therefore, there is an urgent need to understand the mechanisms that suppress the growth of interfacial instabilities. In this talk, I introduce our laser astrophysics experiments and theoretical understandings on the amplification of magnetic fields by the Richtmyer-Meshkov instability [1-3].

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Rigidity of Beltrami fields with a non-constant proportionality factor

K. Abe (Osaka City U.)

Beltrami fields $\operatorname{curl} u = fu$, $\operatorname{div} u = 0$ appear as steady states of ideal incompressible flows or plasma equilibria. I will discuss existence and non-existence issues on them for non-constant factor f. In the first half of the talk, I will explain existence of axisymmetric Beltrami fields forming vortex rings and their construction via a variational principle. In the second half, I will discuss a rigidity problem on symmetry of u for symmetric f and a relation with Grad's conjecture. This talk is based on preprints arXiv:2008.09345, arXiv:2108.03870.

Exact solutions in non-integrable systems

C. Matsuoka (Osaka City U.)
K. Hiraide (Ehime U.)

We present exact solutions in non-integrable systems, taking the Henon map as an example. The solutions are constructed using the Laplace transform. Unlike integrable systems such as soliton systems, the obtained solutions cannot be described by any existing functions. We visualize a boundary of KAM tori and the Henon attractor by use of the obtained functions, in which chaotic orbits are captured accurately.

Dirac geometry and variational structures in nonequilibrium thermodynamics

H. Yoshimura (Waseda U.)

A Dirac structure is a geometric object of unifying symplectic and Poisson structures. In mechanics, for the case of degenerate Lagrangian systems with nonholonomic constraints, which are linear in velocities, given as a distribution on a configuration manifold Q, it is known that an induced (almost) Dirac structure over the Pontryagin bundle $P = TQ \oplus T^*Q$ can be defined from the given distribution.

On the other hand, a general class of nonholonomic systems with nonlinear constraints may be formulated by a general Lagrange-d'Alembert principle by introducing variational constraints $C_V \subset TQ \oplus T^*Q$ as well as nonlinear kinematic constraints $C_K \subset TQ$, in which case one cannot define any appropriate geometric structure such as Poisson, symplectic as well as Dirac structures for such a general class of nonholonomic systems with nonlinear constraints.

In this talk, we study a class of nonholonomic systems with nonlinear onstraints of thermodynamic type, in which C_K can be mathematically deduced from C_V. For the cases of nonholonomic systems with nonlinear constraints of thermodynamic type, we show that an induced Dirac structure $D_P \subset TP \oplus T^*P$ can be defined from C_V and therefore a Dirac dynamical system can be introduced by using D_P. We also show that there exists an associated variational structure with the Dirac dynamical system. In thermodynamics, such a class of nonlinear nonholonomic constraints typically appears in the relation of entropy production regarding the second law of thermodynamics. Finally, we show that We illustrate our theory with some examples of nonequilibrium thermodynamic systems for the case of adiabatically closed and open cases.

This is a joint work with François Gay-Balmaz in École Normale Supérieure in Paris.

Continuous symmetries of semi-discrete equations and conserved quantities

L. Peng (Keio U.)

During the1880s, Sophus Lie started his study on what role continuous transformations play in the solvability of ordinary differential equations. Nowadays, we know it well that solvability or integrability of differential equations is greatly due to the existence of sufficient symmetries, i.e., continuous transformations moving one solution to another solution, or sufficient conserved quantities. For variational equations, they are connected to each other according to the celebrated Noether's theorem in a one-to-one manner.

On the other hand, many mechanical and physical systems can be modeled as semi-discrete equations, such the Toda lattice. However, symmetries of semi-discrete equations have not been well established because prolonging a transformation to variables involving both derivatives and shifts has been the essential challenge, that has bothered scholars for decades. In this talk, we will show how this problem is eventually solved that allows us to extend Noether's two theorems to semi-discrete equations.

Particle relabeling symmetry and Noether's first and second theorems for fluid and MHD equations

Y. Fukumoto (Kyushu U.)
R. Zou (U. Hawai'i)

For the ideal fluid dynamics and magnetohydrodynamics (MHD), Noether's theorem states that the topological invariant associated with the particle relabeling symmetry is the cross helicity, the volume integral of the scalar product of the velocity field and a frozen-in field. A proof to it is given in terms of variation of the Lagrangian label as a function of the Eulerian position. According to Noether's second theorem, the existence of the topological invariant implies redundancy of the governing equations. The generalized Bianchi identity reflecting it is derived for MHD.